Blok 1 - Vaardigheden

bladzijde 52

1a $3^{2+3x} = \frac{1}{3}$
$3^{2+3x} = 3^{-1}$
$2+3x = -1$
$3x = -3$ dus $x = -1$

b $8 \cdot (\sqrt{2})^x = \frac{1}{2}$
$2^3 \cdot (2^{\frac{1}{2}})^x = 2^{-1}$
$2^{3+\frac{1}{2}x} = 2^{-1}$
$3 + \frac{1}{2}x = -1$
$\frac{1}{2}x = -4$ dus $x = -8$

c $7^{4x} \cdot 7^{1-2x} = 1$
$7^{2x+1} = 7^0$
$2x+1 = 0$
$2x = -1$ dus $x = -\frac{1}{2}$

d $2\sqrt{6} \cdot 36^x = 12$
$\sqrt{6} \cdot 36^x = 6$
$6^{\frac{1}{2}} \cdot (6^2)^x = 6^1$
$\frac{1}{2} + 2x = 1$
$2x = \frac{1}{2}$ dus $x = \frac{1}{4}$

e $4 \cdot 2^{3x+7} = 8 \cdot 4^x$
$2^2 \cdot 2^{3x+7} = 2^3 \cdot (2^2)^x$
$2^{3x+9} = 2^{3+2x}$
$3x+9 = 3+2x$
$x = -6$

2a $3200 + 25 \cdot 1,2^t = 5000$
$25 \cdot 1,2^t = 1800$
$1,2^t = 72$
$t = {}^{1,2}\log 72$
$t = \dfrac{\log 72}{\log 1,2} \approx 23,457$

b $1875 - 50 \cdot 0,95^t = 1865$
$50 \cdot 0,95^t = 10$
$0,95^t = \frac{1}{5}$
$t = {}^{0,95}\log \frac{1}{5}$
$t = \dfrac{\log \frac{1}{5}}{\log 0,95} \approx 31,377$

c $150 \cdot 0,45^{t-6} = 6$
$0,45^{t-6} = \frac{1}{25}$
$t - 6 = {}^{0,45}\log \frac{1}{25}$
$t = 6 + {}^{0,45}\log \frac{1}{25}$
$t = 6 + \dfrac{\log \frac{1}{25}}{\log 0,45} \approx 10,031$

d $0,04 + 0,25 \cdot 3,5^{t+10} = 1$
$0,25 \cdot 3,5^{t+10} = 0,96$
$3,5^{t+10} = 3,84$
$t + 10 = {}^{3,5}\log 3,84$
$t = {}^{3,5}\log 3,84 - 10$
$t = \dfrac{\log 3,84}{\log 3,5} - 10 \approx -8,926$

3a $u = 0,5 \cdot 3^p$
$3^p = 2u$
$p = {}^{3}\log 2u$

b $u = 1 - 2 \cdot 0,65^p$
$2 \cdot 0,65^p = 1 - u$
$0,65^p = \frac{1}{2} - \frac{1}{2}u$
$p = {}^{0,65}\log(\frac{1}{2} - \frac{1}{2}u)$

c $10u - 1,2^p = 250$
$1,2^p = 10u - 250$
$p = {}^{1,2}\log(10u - 250)$

d $u - 400 \cdot 0,5^p = 1000$
$400 \cdot 0,5^p = u - 1000$
$0,5^p = \frac{1}{400}u - 2\frac{1}{2}$
$p = {}^{0,5}\log\left(\frac{1}{400}u - 2\frac{1}{2}\right)$

bladzijde 53

4a $V = \frac{5}{p+1} - 8$

$V + 8 = \frac{5}{p+1}$ dus $(V+8)(p+1) = 5$

b $p + 1 = \frac{5}{V+8}$ geeft $p = \frac{5}{V+8} - 1$

5a $N = \frac{15}{250 + t}$

$250 + t = \frac{15}{N}$

$t = \frac{15}{N} - 250$

b $N = \frac{-30}{2t - 30}$

$2t - 30 = \frac{-30}{N}$

$2t = \frac{-30}{N} + 30$

$t = \frac{-15}{N} + 15$

c $N = 3 + \frac{50}{10 + 2t}$

$N - 3 = \frac{50}{10 + 2t}$

$10 + 2t = \frac{50}{N - 3}$

$2t = \frac{50}{N - 3} - 10$

$t = \frac{25}{N - 3} - 5$

d $N = \frac{3}{1 + 2^t}$

$1 + 2^t = \frac{3}{N}$

$2^t = \frac{3}{N} - 1$

$t = {}^{2}\log\left(\frac{3}{N} - 1\right)$

e $N = \frac{24}{2 + 0{,}5 \cdot 3^t}$

$2 + 0{,}5 \cdot 3^t = \frac{24}{N}$

$0{,}5 \cdot 3^t = \frac{24}{N} - 2$

$3^t = \frac{48}{N} - 4$

$t = {}^{3}\log\left(\frac{48}{N} - 4\right)$

f $N = 10 + \frac{1}{0{,}8^t}$

$N - 10 = \frac{1}{0{,}8^t}$

$0{,}8^t = \frac{1}{N - 10}$

$t = {}^{0{,}8}\log\frac{1}{N - 10}$

6a $\frac{300}{1 + 299 \cdot 0{,}65^t} = 250$

$1 + 299 \cdot 0{,}65^t = \frac{300}{250}$

$299 \cdot 0{,}65^t = \frac{300}{250} - 1$

$299 \cdot 0{,}65^t = 0{,}2$

$0{,}65^t = \frac{0{,}2}{299}$

$t = {}^{0{,}65}\log\left(\frac{0{,}2}{299}\right) = \frac{\log\left(\frac{0{,}2}{299}\right)}{\log 0{,}65}$

$t \approx 16{,}97$

Na ongeveer 17 weken heeft de plant als lengte 250 cm.

b $\frac{300}{1 + 299 \cdot 0{,}65^t} = 150$

$1 + 299 \cdot 0{,}65^t = 2$

$299 \cdot 0{,}65^t = 1$

$0{,}65^t = \frac{1}{299}$

$t = {}^{0{,}65}\log\left(\frac{1}{299}\right) = \frac{\log\left(\frac{1}{299}\right)}{\log 0{,}65}$

$t \approx 13{,}23$

Na ongeveer 13 weken neemt de groeisnelheid niet meer toe.

c

t	0	1	2	3	4	5
L	1	1,536	2,356	3,610	5,517	8,405

$\frac{1,536}{1} \approx \frac{2,356}{1,536} \approx \frac{3,610}{2,356} \approx \frac{5,517}{3,610} \approx \frac{8,405}{5,517} \approx 1,53$

De factor is ongeveer 1,53.

d $L(4) - L(3) \approx 5,52 - 3,61 = 1,91$ cm

e Voor grote waarden van t nadert $0,65^t$ naar nul dus nadert L naar $\frac{300}{1+299 \cdot 0} = 300$ cm dus 3 m.

f $L = \dfrac{300}{1 + 299 \cdot 0,65^t}$

$1 + 299 \cdot 0,65^t = \dfrac{300}{L}$

$299 \cdot 0,65^t = \dfrac{300}{L} - 1$

$0,65^t = \dfrac{\frac{300}{L} - 1}{299} = \dfrac{300}{299L} - \frac{1}{299}$

$t = {}^{0,65}\log\left(\frac{300}{299L} - \frac{1}{299}\right)$

g $L = 100$ geeft $t = {}^{0,65}\log\left(\frac{2}{299}\right) \approx 11,6237$

$L = 200$ geeft $t = {}^{0,65}\log\left(\frac{1}{299 \cdot 2}\right) \approx 14,8418$

In ongeveer 3,22 weken groeit de plant van 1 tot 2 meter.

bladzijde 54

7a $21 + 6 \cdot {}^{2}\log(x-8) = 24$

$6 \cdot {}^{2}\log(x-8) = 3$

${}^{2}\log(x-8) = \frac{1}{2}$

$x - 8 = 2^{\frac{1}{2}}$

$x = \sqrt{2} + 8 \approx 9,414$

b $35 - 2 \cdot {}^{3}\log 4p = 32$

$2 \cdot {}^{3}\log 4p = 3$

${}^{3}\log 4p = 1\frac{1}{2}$

$4p = 3^{1\frac{1}{2}}$

$p = \frac{1}{4} \cdot 3 \cdot 3^{\frac{1}{2}} = \frac{3}{4}\sqrt{3} \approx 1,299$

c ${}^{6}\log 7 + {}^{6}\log 2x = 1$

${}^{6}\log 14x = 1$

$14x = 6^1$

$x = \frac{6}{14} = \frac{3}{7} \approx 0,429$

d $\log(3x - 17) - \log 4 = 2$

$\log(\frac{3}{4}x - 4\frac{1}{4}) = 2$

$\frac{3}{4}x - 4\frac{1}{4} = 10^2$

$\frac{3}{4}x = 104\frac{1}{4}$

$x = 139$

8a $P = 8K + {}^{2}\log 1024 + 40$

$P = 8K + 50$

$8K = P - 50$

$K = \frac{1}{8}P - 6\frac{1}{4}$

b $P = 200 + {}^{2}\log W + 40$

${}^{2}\log W = P - 240$

$W = 2^{P-240}$

c $50 = 8K +^2 \log W + 40$

$^2\log W = 10 - 8K$

$W = 2^{10-8K}$

9a $r = 24 + \sqrt{5t+24} - 8$

$\sqrt{5t+24} = r - 16$

$5t + 24 = (r-16)^2$

$5t = (r-16)^2 - 24$

$t = \frac{1}{5}(r-16)^2 - 4\frac{4}{5}$

b $r = 3d + \sqrt{144} - 8$

$r = 3d + 4$

$3d = r - 4$

$d = \frac{1}{3}r - 1\frac{1}{3}$

c $0 = 3d + \sqrt{5t+24} - 8$

$\sqrt{5t+24} = 8 - 3d$

$5t + 24 = (8-3d)^2$

$5t = (8-3d)^2 - 24$

$t = \frac{1}{5}(8-3d)^2 - 4\frac{4}{5}$

10a $53,3 = 20 \cdot \log N + \frac{4}{3} \cdot 76 - 157$

$20 \cdot \log N = 108\frac{29}{30}$

$\log N = 5\frac{269}{600}$

$N = 10^{5\frac{269}{600}} \approx 280\,759$ vliegtuigen per jaar.

b $B = 20 \cdot \log(2N) + \frac{4}{3}L - 157$

$= 20 \cdot (\log 2 + \log N) + \frac{4}{3}L - 157$

$= 20 \cdot \log 2 + 20 \log N + \frac{4}{3}L - 157$

De geluidsbelasting neemt toe met $20 \log 2$ dus is er geen sprake van verdubbeling.

c $53,3 = 20 \cdot \log N + \frac{4}{3}L - 157$

$\frac{4}{3}L = 210,3 - 20 \log N$

$L = 157,725 - 15 \log N$

d $45 = 20 \log N + \frac{4}{3}L - 157$

$20 \log N = 202 - \frac{4}{3}L$

$\log N = 10,1 - \frac{1}{15}L$

$N = 10^{10,1-\frac{1}{15}L} = 10^{10,1} \cdot 10^{-\frac{1}{15}L}$

$= 10^{10,1} \cdot \left(10^{-\frac{1}{15}}\right)^L \approx 1,259 \cdot 10^{10} - 0,858^L$

e $N(76) = 1,259 \cdot 10^{10} \cdot 0,858^{76} \approx 110\,933$ en

$N(70) = 1,259 \cdot 10^{10} \cdot 0,858^{70} \approx 278\,058$

Er mogen $278\,058 - 110\,933 = 167\ 125$ vliegtuigen meer vliegen.

bladzijde 55

11a $200t^5 = 4500$

$t^5 = 22,5$

$t = 22,5^{\frac{1}{5}} \approx 1,864$

b $64 \cdot p^4 = 640$

$p^4 = 10$

$p = -10^{\frac{1}{4}} \approx -1,778$ of

$p = 10^{\frac{1}{4}} \approx 1,778$

c $0,2 + 0,1 \cdot x^{15} = 0,6$

$0,1 \cdot x^{15} = 0,4$

$x^{15} = 4$

$x = 4^{\frac{1}{15}} \approx 1,097$

d $1 + 0,23d^{0,63} = 3,8$

$0,23d^{0,63} = 2,8$

$d^{0,63} = \dfrac{2,8}{0,23}$

$d = \left(\frac{2,8}{0,23}\right)^{\frac{1}{0,63}} \approx 52,833$

e $2000 - 12 \cdot a^{-3,1} = 1100$

$12 \cdot a^{-3,1} = 900$

$a^{-3,1} = 75$

$a = 75^{\frac{1}{-3,1}} \approx 0,248$

f $432 + 3 \cdot x^6 = 132$

$3 \cdot x^6 = -300$

$x^6 = -100$

Geen oplossingen.

12a $L = 10 \cdot K^{0,5}$

$K^{0,5} = \frac{1}{10} L$

$K = \frac{1}{100} L^2$

b $L = 10 + K^{-5}$

$K^{-5} = L - 10$

$K = (L - 10)^{-\frac{1}{5}}$

c $L = (3 - 2K)^3$

$3 - 2K = L^{\frac{1}{3}}$

$2K = 3 - L^{\frac{1}{3}}$

$K = 1\frac{1}{2} - \frac{1}{2} L^{\frac{1}{3}}$

d $L^2 \cdot K^{2,5} = 25$

$K^{2,5} = 25 \cdot L^{-2}$

$K = 25^{\frac{1}{2,5}} \cdot L^{-\frac{2}{2,5}}$

13a $5p^2 - 3p = 0$

$p(5p - 3) = 0$

$p = 0$ of $p = \frac{3}{5}$

b $(a^n)^m = a^{nm}$ dus $(2^x)^2 = 2^{2x}$

c $2^x = 0$ heeft geen oplossing

d $2^x = \frac{3}{5}$ geeft $x = {}^2\log 0,6$

14a $3^{2x} - 5 \cdot 3^x + 6 = 0$

$(3^x)^2 - 5 \cdot 3^x + 6 = 0$ (Stel $p = 3^x$)

$p^2 - 5p + 6 = 0$

b $p^2 - 5p + 6 = 0$

$(p - 2)(p - 3) = 0$

$p = 2$ of $p = 3$

c $3^x = 2$ of $3^x = 3$

$x = {}^3\log 2$ of $x = 1$

15a $3^{2x} - 4 \cdot 3^x - 5 = 0$ (Stel $p = 3^x$)

$p^2 - 4p - 5 = 0$

$(p+1)(p-5) = 0$

$p = -1$ of $p = 5$

$3^x = -1$ of $3^x = 5$

$x = {}^3\log 5$

b $4^{2x} + 4^x - 12 = 0$ (Stel $p = 4^x$)

$p^2 + p - 12 = 0$

$(p+4)(p-3) = 0$

$p = -4$ of $p = 3$

$4^x = -4$ of $4^x = 3$

$x = {}^4\log 3$

c $(2^x - 2\sqrt{2})(2^x - \frac{1}{8}) = 0$

$2^x - 2\sqrt{2} = 0$ of $2^x - \frac{1}{8} = 0$

$2^x = 2\sqrt{2}$ of $2^x = \frac{1}{8}$

$2^x = 2^{1\frac{1}{2}}$ of $2^x = 2^{-3}$

$x = 1\frac{1}{2}$ of $x = -3$

d $5^{2x} - 20 \cdot 5^x = 125$ (Stel $p = 5^x$)

$p^2 - 20p - 125 = 0$

$(p-25)(p+5) = 0$

$p = 25$ of $p = -5$

$5^x = 25$ of $5^x = -5$

$x = 2$

e $(4^x - 1)(4^x + 1) = 27$

$4^{2x} - 1 = 27$

$4^{2x} = 28$

$2x = {}^4\log 28$

$x = \frac{1}{2} \cdot {}^4\log 28 = {}^4\log \sqrt{28}$

f $2^x(2^x - 1) = 32 \cdot (2^x - 1)$

$2^x - 1 = 0$ of $2^x = 32$

$2^x = 1$ of $2^x = 2^5$

$x = 0$ of $x = 5$